Differentiation Technique - Quotient Rule

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**Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).**

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Particular Solution of Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Very Hard / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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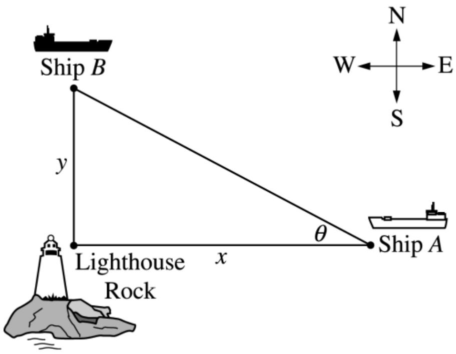
Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Related Rates, Implicit Differentiation, Modelling Situations, Differentiation Technique – Standard Functions, Differentiation Technique – Trigonometry, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Hard / Question Number: 6



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

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Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule, Points Of Inflection, Calculating Limits Algebraically, Tangents To Curves

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 6

6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

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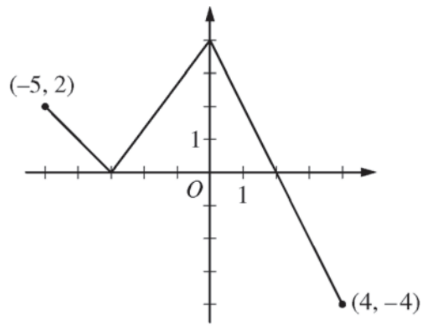
Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Increasing/Decreasing , Concavity, Differentiation Technique - Quotient Rule, Tangents To Curves, Differentiation Technique – Chain Rule, Integration Graphs

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 3



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

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Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Tangents To Curves, Vertical Tangents, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Somewhat Challenging / Question Number: 6

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

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Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique - Quotient Rule, Fundamental Theorem of Calculus (First), Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Implicit Differentiation, Differentiation Technique – Trigonometry, Differentiation Technique – Product Rule, Tangents To Curves, Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Hard / Question Number: 5

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

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Question 8

Qualification: AP Calculus AB

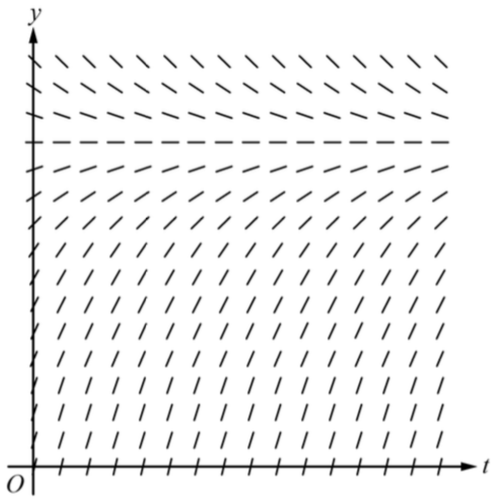
Areas: Applications of Differentiation, Differentiation, Differential Equations

Subtopics: Sketching Slope Field, Interpreting Meaning in Applied Contexts, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Increasing/Decreasing , Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Medium / Question Number: 6

6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t + 2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

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